

Cambridge International AS & A Level

CANDIDATE NAME					
CENTRE NUMBER			CANDIDATE NUMBER		

0 1 2 3 4 5 6 7 8 9

FURTHER MATHEMATICS

9231/01

Paper 1 Further Pure Mathematics 1

For examination from 2020

SPECIMEN PAPER 2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do not write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has 16 pages. Blank pages are indicated.

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		2	
1	(a)	Given that $f(r) = \frac{1}{(r+1)(r+2)}$, show that	
		$f(r-1) - f(r) = \frac{2}{r(r+1)(r+2)}$.	[2]
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			••••
			••••
	(b)	Hence find $\sum_{r=1}^{n} \frac{1}{r(r+1)(r+2)}.$	[3]
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			••••

(c)	Deduce the value of $\sum_{r=1}^{\infty} \frac{1}{r(r+1)(r+2)}.$	[1]
		••••

Prove,	by mathem	natical indu	action, th	at $\phi(n)$ is	s divisibl	e by 8 fo	r every p	ositive int	eger n.	
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[3]

The curve *C* has polar equation $r = 2 + 2 \cos \theta$, for $0 \le \theta \le \pi$.

3

(a) Sketch C.

Find the area	of the region encl	losed by C and th	e initial line.	
Find the area	of the region encl	losed by C and th	e initial line.	
Find the area	of the region encl	losed by C and th	e initial line.	
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Find the area		losed by C and th	e initial line.	
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		losed by C and th	e initial line.	

(c)	Show that the Cartesian equation of C can be expressed as $4(x^2 + y^2) = (x^2 + y^2 - 2x)^2$. [3]

4 The cubic equation

$$z^3 - z^2 - z - 5 = 0$$

has roots α , β and γ .

Show that the value of $\alpha^3 + \beta^3 + \gamma^3$ is 19.	

(b)	Find the value of $\alpha^4 + \beta^4 + \gamma^4$.	2]
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(c)	Find a cubic equation with roots $\alpha + 1$, $\beta + 1$ and $\gamma + 1$, giving your answer in the form	
	$px^3 + qx^2 + rx + s = 0,$	
	where p , q , r and s are constants to be determined.	3]
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5	The	matrix	A	is	given	hv
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$$\mathbf{A} = \begin{pmatrix} 5 & k \\ -3 & -4 \end{pmatrix}$$

(a)	Find the value of k for which A is singular.	[2]
	(5 6)	
It is	s now given that $k = 6$ so that $\mathbf{A} = \begin{pmatrix} 5 & 6 \\ -3 & -4 \end{pmatrix}$.	
(b)	Find the equations of the invariant lines, through the origin, of the transformation represented by A .	in the <i>x-y</i> plane [6]
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(c)	The	e triangle DEF in the x-y plane is transformed by A onto triangle PQR .	
` /			
		Given that the area of triangle DEF is $10 \mathrm{cm}^2$, find the area of triangle PQR .	[2]
			[2]
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	(i)	Given that the area of triangle <i>DEF</i> is 10 cm ² , find the area of triangle <i>PQR</i> .	
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6	The position	vectors	of the	points A	. B.	C.D	are

$$2i + 4j - 3k$$
, $-2i + 5j - 4k$, $i + 4j + k$, $i + 5j + mk$,

respectively, where m is an integer. It is given that the shortest distance between the line through A and B and the line through C and D is B.

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Find the shortes	st distance of D fi	rom the line thre	ough A and C .		
Find the shortes	st distance of <i>D</i> fr	rom the line thr	ough A and C.		
Find the shortes	st distance of <i>D</i> fi	rom the line thr	ough A and C.		
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Find the shortes	st distance of <i>D</i> fi	rom the line thr	ough A and C.		
	st distance of <i>D</i> fr				

(c)	Show that the acute angle between the planes ACD and BCD is $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$.	[4]
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7	The	the curve C has equation $y = \frac{2x^2 - 3x - 2}{x^2 - 2x + 1}$.	
	(a)	State the equations of the asymptotes of <i>C</i> .	[2]
	(b)	Show that $y \leqslant \frac{25}{12}$ at all points on C .	[4
			•••••

(c)	Find the coordinates of any stationary points of <i>C</i> .	[3]
(d)	Sketch C , stating the coordinates of any intersections of C with the coordinate axes asymptotes.	and the [4]

(e)	Sketch the curve with equation $y = \left \frac{2x^2 - 3x - 2}{x^2 - 2x + 1} \right $ and find the set of values of x for which
	$\left \frac{2x^2 - 3x - 2}{x^2 - 2x + 1} \right < 2. \tag{4}$

Additional page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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